3.4 Trigonometric Proof

Question Paper

Course	CIE AS Maths
Section	3. Trigonometry
Topic	3.4 Trigonometric Proof
Difficulty	Very Hard

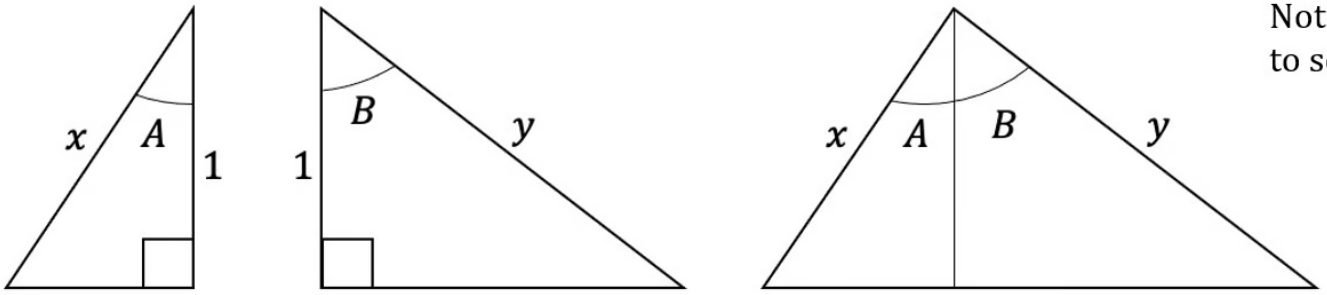
Time allowed: 60

Score: /48

Percentage: /100

Question 1

Consider the three triangles, all of height 1, as shown below.



Not to scale

By applying the area of a triangle formula $A = \frac{1}{2}ab\sin C$ to each one, prove that,

$$\sin(A + B) \equiv \sin A \cos B + \sin B \cos A$$

Briefly explain why this only proves the result for A and B being acute angles.

[6 marks]

Question 2

Prove the identity

$$\tan 4\theta \equiv \frac{4\tan\theta(1-\tan^2\theta)}{1-6\tan^2\theta+\tan^4\theta}$$

[4 marks]

Question 3

Prove the identity

$$-16\cot 2\theta\,\operatorname{cosec}^3 2\theta \equiv \sec^4\theta - \operatorname{cosec}^4\theta$$

[5 marks]

Question 4

Show that

$$\frac{\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right)}{\sin\left(\theta-\frac{\pi}{2}\right)} \equiv \tan\theta - 1$$

[4 marks]

Question 5a

(a) Show that

$$\sin 3\theta \equiv 3\sin\theta\cos^2\theta - \sin^3\theta$$

[4 marks]

Question 5b

(b) Hence, or otherwise, show that

$$\frac{\cos 3\theta - \cos \theta}{\sin 3\theta \sin \theta} \equiv \frac{4 \cos \theta}{1 - 4 \cos^2 \theta} \qquad \theta \neq k\pi$$

[5 marks]

Question 6

Show that

$$4\cos^2\left(x - \frac{\pi}{6}\right) \equiv 3 - 2\sin^2 x + \sqrt{3}\sin 2x$$

[5 marks]

Question 7

Show that

$$\tan\left(\frac{2x+\pi}{4}\right) \equiv \sec x + \tan x$$

[6 marks]

Question 8

Show that

$$\frac{1}{\left(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta\right)^2} \equiv 4\,\mathrm{cosec}^2\left(2\theta + \frac{\pi}{3}\right)$$

[9 marks]